LA-UR-92- - 3165

Title: AN INTENSE SOURCE OF POSITRONS USING A LOW ENERGY PROTON BEAM

LA-UR--92-3165

DE93 000697

Author(s): Paul J. Channell and Harry Dreicer

Submitted to: To be published in the Proceedings of the Workshop on Advanced Accelerator Concepts
June 14-20, 1992
Port Jefferson, NY

Los Alamos
NATIONAL LABORATORY

An Intense Source of Positrons Using a Low Energy Proton Beam

Paul J. Channell, AT-7, MS H829
Harry Dreicer, ADRE, MS A104
Los Alamos National Laboratory
Los Alamos, NM 87545

1 Introduction

An intense source of positrons (say $10^{14}e^+$ per second or more) would make it possible to consider linear collider designs for the flavor factories (ϕ and B) now being proposed. Such designs might have the advantage of higher luminosity than circular colliders and would also provide an arena for the further development of linear colliders in preparation for the next generation of TeV linear colliders. An intense source of positrons would also make it possible to be more flexible in the design of the next generation of TeV linear colliders.

The two conventional sources of positrons now used are electron-beam electroproduction of positrons and reactor-produced β^+-unstable isotopes. Electroproduction sources can be fairly efficient (one e^+ per 30-GeV electron used), but suffer from the disadvantages of high capital cost (a multi-GeV electron accelerator) and destruction of the target at higner fluxes. Reactor sources are much more inefficient and have much higher capital costs than electroproduction sources, but might make sense if a reactor already exists at an appropriate site and is principally used for other purposes.

In this paper we will show that it should be possible to produce large fluxes of positrons using intense proton beams (100 mA CW) of modest energy (6 – 10 MeV). The basic idea is to use protons from an accelerator rather than neutrons from a reactor to produce β^+-unstable isotopes by the proton capture reaction. Because the targets of a proton beam can be handled more conveniently and much more quickly than insertions in reactors, we are able to consider a larger range of possible targets with shorter decay times than is reasonable for a reactor. Because an accelerator beam is better defined and more easily controlled than neutrons in a reactor, the efficiency of conversion of beam energy into positrons can be much larger than in a reactor. Because the β^+-unstable isotopes from the target of the proton beam can be quickly transferred into a magnetic trap, after various combinations of chemical and mechanical separation of the β^+-unstable isotopes from the majority target species (in less than a decay time), positron annihilation can be largely avoided and the efficiency of useful positron production can be as high as that of the electroproduction sources.

In the next section, we will discuss the proton beam production of β^+-unstable isotopes and show that one can produce as many as $10^{14}e^+$ per second using 100-mA CW proton beams with energies of $6-10$ MeV. There are a number of possible targets, all with cross sections large enough that approximately one in 10^3 protons will be converted into β^+-unstable isotopes. Of course, once the positrons are produced it is necessary to avoid positron annihilation on background electrons and to capture the positrons in a small enough region of phase space to be useful, i.e., small enough that the positrons can be subsequently accelerated and cooled in a damping ring. Because only a very small fraction (≈ 1 in 10^8) of the target nuclei undergo proton capture, it is essential to separate the β^+-unstable isotopes from the majority target species in order to avoid annihilation. After various combinations of chemical and mechanical separation, the β^+-unstable isotopes can be easily trapped in a magnetic bottle (e.g. a mirror, a cusp, etc.) where they decay into other nuclei and positrons. To produce ultracold positrons one could consider long-time confinement and radiative cooling in a magnetic trap (possibly a mirror or a cusp with electrostatic confinement added), although for large β^+ fluxes the resulting trap sizes are likely to be quite large. In section 3, we will consider a more modest trap, a mirror, which can be analyzed straightforwardly and which is good enough to provide radiative cooling of the multi-MeV positrons by a factor of about 4. The resulting output beam from the mirror could be accelerated to about 5 MeV and then passed through a foil cooling array, with small attendant annihilation, to produce a beam with a normalized emittance of about 2000π mm mrad, i.e., low enough that it can then be accelerated in a reasonable linac to an energy suitable for injection into a damping ring. We will discuss the foil cooling in section 4. In the final section we will summarize our results and briefly discuss variations one might consider.

2 Targets and Beam Requirements

In order to produce β^+-unstable isotopes using proton beams we must have a beam energy larger than the Coulomb barrier but smaller than the threshold energy for neutron production. We have not made an exhaustive study of all possible targets; however, based on the requirements that proton capture yield a β^+-unstable isotope, and that the target isotope be the most abundant form of the element, we have identified five candidate targets; the targets, the proton capture reactions, and the resulting decays are

$$p +^{14} N \rightarrow^{15} O + \gamma \rightarrow^{15} N + e^+,$$

$$p + M^{24}g \rightarrow^{25} Al + \gamma \rightarrow^{25} Mg + e^+,$$

$$p +^{28} Si \rightarrow^{29} P + \gamma \rightarrow^{29} Si + e^+,$$

$$p +^{32} S \rightarrow^{33} Cl + \gamma \rightarrow^{33} S + e^+,$$

$$p +^{40} Ca \rightarrow^{41} Sc + \gamma \rightarrow^{41} Ca + e^+,$$

where the γ in the intermediate state can stand for one or more prompt or decay gammas. For all of these reactions the (p, n), (p, α), $(p, 2n)$, etc. channels all have Q-values that are negative with magnitudes at least several MeV higher than the Coulomb barrier. The Coulomb barriers (i.e., the minimum proton beam energy required), the decay lifetimes for the second step, and the resulting positron energies for these reactions are shown in Table 1.

Table 1

Target	Coulomb Barrier (MeV)	Decay (sec)	e^+ Energy (MeV)
^{14}N	3.2	124.0	1.74
^{24}Mg	4.6	7.2	3.2
^{28}Si	5.1	4.4	3.95
^{32}S	5.6	2.5	4.5
^{40}Ca	6.5	0.55	5.6

The target thicknesses are limited by the energy loss of the proton beam. For nonrelativistic protons the energy loss formula [1] is approximately

$$\frac{dE}{dz} = \frac{2\pi n Z e^4}{E}\left(\frac{M}{m}\right)\ln\left(\frac{4mE}{\hbar < \omega > M}\right),$$

where E is the energy of the proton, n is the density of the target, Z is the number of protons in the target nucleus, m is the electron mass, M is the proton mass, and $\hbar < \omega >$ is an average excitation energy of the target atoms. Let us define the loss distance to be

$$L \equiv \left(\frac{1}{E}\frac{dE}{dz}\right)^{-1}.$$

The computed loss distances for the proton beams (taking the energy to be 3 MeV over the Coulomb barrier) is given in Table 2.

Target	Loss Distance (cm)
N^{14} (100 atm)	0.9
Mg^{24}	0.056
Si^{28}	0.048
S^{32}	0.061
Ca^{40}	0.096

Table 2

The thickness of the target cannot be greater than the loss distances given in this table lest the proton beam energy fall significantly below the Coulomb barrier. To estimate the efficiency of the conversion of protons to positrons, we use a target thickness equal to the proton loss distance and assume the capture cross section is a constant $\sigma = 350$ mb. Of course, the actual cross sections are slightly different from this for each target and vary as the protons slow down. The value we have assumed is typical of the cross section above the Coulomb barrier. The actual cross sections are somewhat higher at the initial beam energy and decrease to slightly less than this value as the beam loses energy. The proton conversion probability then turns out to be about the same in all five cases and is about 8.4×10^{-4}.

Note that the target will be vaporized (if solid) by the beam energy deposition, but this is necessary in any case for subsequent injection into a magnetic trap. One can envision various types of target arrangements, from streams of millimeter-sized pellets fired across the proton beam to flowing jets of gas or prevaporized target material at high pressure.

It is interesting to use the efficiency of conversion to determine the proton current required for the most demanding application, a very high luminosity

B-factory. It would probably be possible to design a linear collider B-factory with a luminosity of $1.3 * 10^{34}$ cm^{-2}sec^{-1} if we had a source of $7 * 10^{14}$ positrons per second [2]. If we assume that all the positrons produced can be trapped, cooled, and extracted, then the above-calculated efficiency can produce $7 * 10^{14}$ positrons per second if the proton beam has a CW current of 130 mA. Using a more realistic overall capture and cooling efficiency of 40%, we see that we need about 325 mA of proton current for the very high luminosity B-factory. Designs based on lower luminosity, of course, require less proton current.

In order to avoid excess annihilation we must separate the β^+-unstable isotopes from the untransmuted isotopes. Because the β^+-unstable isotopes are a different chemical species from the background target material, this should be relatively easy. Consider, for example, the first reaction, transmuting nitrogen to oxygen. It should be possible to devise chemical reactions, possibly followed by mechanical transfer pumping, that will separate these two with high efficiency in the time available (about two minutes). For the other reactions listed the chemical differences are not as great but could be large enough to allow efficient separation techniques.

Of course, other, more exotic, separation techniques can be considered if necessary. For example, because of the relatively large mass differences in the stable and unstable isotopes ($\approx 5\%$), electromagnetic separation of the isotopes should be fast and efficient. If it is necessary to achieve nearly 100% separation, one might consider laser separation by selective ionization of the unstable isotopes; this could be combined with injection into a magnetic trap by ionizing the unstable isotopes as they cross the magnetic bottle and pumping the background un-ionized gas. This technique would, of course, require some laser development, but is probably unnecessary, as ordinary chemical separation should achieve separation efficiencies $greaterthan 50\%$.

3 Traps

Once the β^+-unstable isotopes have been separated from the background target nuclei, they can be magnetically trapped, say by ionization of gas or vapor in the region of the trap, where they decay and where the positrons are "born" trapped [3]. (Note that having the positrons created in the trap is not a practical alternative for an electroproduction source because of the difficulty of injecting the electron beam into the trap.) It might be possible to make a trap, possibly a spindle cusp with electrostatic fields along the field lines at the field maxima, with very good confinement so that the positrons radiatively cool to very low temperatures. However, such a trap would, in all probability, be quite large and its confinement properties would be untested and thus quite uncertain. In this paper we will consider, instead, a conceptually simpler trap, a magnetic mirror.

Though we can not compute the output distributions from a trap as complex as a cusp, it is straightforward to compute the expected output from a magnetic mirror; as we will see, the performance of a simple mirror is good enough that it should be considered a strong candidate for the positron trap.

3.1 Positron Trapping

In this paper we will use the simplest possible model of a mirror, one in which the particle confinement is determined only by the mirror ratio, i.e., the ratio of maximum magnetic field along a field line to the minimum magnetic field along the line. We will ignore electrostatic fields, though it should be kept in mind that externally imposed electrostatic fields can probably be used to trap positrons for very long times and thus achieve brighter positron beams. We will ignore spatial structure, assuming that the motion along field lines is fast enough that once a particle enters the loss cone it is immediately lost. We have in mind a trap in which the bulk of the plasma is in the low-field region and the end regions occupy only a relatively small volume; the positrons are born from trapped isotopes mostly in the low field region with an energy spectrum given by the Fermi distribution. With these assumptions we can determine whether a particle is trapped or not simply by specifying its position in velocity space.

Our first task is to find the adiabatic invariant for relativistic particles that generalizes the nonrelativistic magnetic moment invariant. This can be computed using the observation that the adiabatic invariant for a periodically oscillating particle is the action integral around the periodic orbit, i.e., the adiabatic invariant, J, is

$$J = \int_0^T \vec{p} \cdot d\vec{q},$$

where T is the period of the oscillation. We let $\vec{p} = \gamma m \vec{v}_\perp$, where $\vec{v}_\perp$ denotes the perpendicular component of velocity. The expression for the period, T, of a particle in a magnetic field is

$$T = \frac{2\pi\gamma mc}{eB},$$

where m is the particle mass, c is the velocity of light, e is the particle charge, and B is the magnitude of the magnetic field. We can now compute J and, multiplying by irrelevant constants to get the right nonrelativistic limit, find that the relativistic magnetic moment is

$$\mu = \frac{\gamma^2 m\vec{v}_\perp^2}{2B}.$$

As the positrons move in the trap, their total energy and their magnetic moment are preserved on time scales short compared with a radiation damping time. Particles with a large enough magnetic moment can not pass over the magnetic field maximum and are trapped. Particles with a small magnetic moment are untrapped and are said to be in the loss cone; they leave the trap almost immediately. In terms of the velocity space of bulk particles, we can compute the location of the loss cone. The Hamiltonian can be expressed as

$$H = \frac{mc^2}{\sqrt{1 - \beta_\|^2 - \frac{2\mu B}{\gamma^2 mc^2}}}.$$

where $\vec{\beta} = \vec{v}/c$. In order for a particle in the bulk of the plasma to just reach the magnetic field maximum with no parallel velocity, energy conservation implies

$$\frac{mc^2}{\sqrt{1 - \beta_\|^2 - \frac{2\mu B_{\rm min}}{\gamma^2 mc^2}}} = \frac{mc^2}{\sqrt{1 - \frac{2\mu B_{\rm max}}{\gamma^2 mc^2}}},$$

where we have used magnetic moment conservation to write the same μ on both sides of this equation. From this equality it is easy to see that a particle will be in the loss cone if

$$\beta_\| \geq \sqrt{R-1}\,\beta_\perp,$$

where $R \equiv B_{\rm max}/B_{\rm min}$ is the mirror ratio. It is interesting to note that this is the same result one gets for nonrelativistic particles.

A positron "born" in the loss cone will leave the trap almost immediately; a positron not in the loss cone will stay in the trap for some time. We will count all positrons born in the loss cone as lost, because even if they do enter the subsequent accelerating sections it is likely that their longitudinal emittance is too large for them to be useful. Let us estimate the fraction of positrons born in the loss cone. The fraction, F, of velocity space occupied by the loss cone is just the total solid angle of the loss cone divided by 4π. Using the fact that the tangent of the angle of the loss cone is $\beta_\perp/\beta_\parallel = 1/\sqrt{R-1}$, it is easy to see that this is just

$$F = 1 - \sqrt{1 - \frac{1}{R}},$$

and that this is well approximated by

$$F \simeq \frac{1}{2R}.$$

As an example, for a mirror ratio $R = 4$, 87% of the positrons will be trapped.

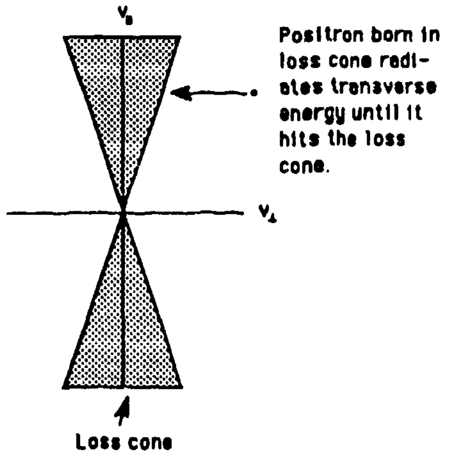

Figure 1.

Most of the positrons that are trapped will eventually become detrapped either by Coulomb collisions on other positrons and background plasma or by radiating their transverse energy away until they fall into the loss cone. For the relatively energetic positrons in the trap, collisional detrapping is negligible. For example, a 0.5-MeV positron in a trap with a density of 10^{12} cm^{-3} will suffer a collision about every 535 seconds and will undergo about

8 of these collisions before being scattered into the loss cone if the mirror ratio is 4; in other words, collisions are negligible on the isotope decay time scale. Thus we can conclude that most of the trapped positrons will be lost after they radiate away their transverse energy and fall into the loss cone, as depicted in Figure 1.

3.2 Radiative Cooling

To compute the confinement time of the positrons and their output distributions we need to compute the effects of radiation. From reference [1], one can show that the radiated power, P, from a particle moving both parallel and transverse to a magnetic field is given by

$$P = \frac{2\beta_\perp^2 \gamma^2 e^4 B^2}{3m^2 c^3}.$$

Using this and the expression for $\beta_\perp$ in terms of $\beta_\parallel$ and γ,we find that γ satisfies the equation

$$\frac{d\gamma}{dt} = -\frac{\gamma^2}{\tau}(1 - \beta_\parallel^2 - \frac{1}{\gamma^2}),$$

where

$$\tau \equiv \frac{3m^3 c^5}{2e^4 B^2}.$$

If we define a variable

$$\xi \equiv \sqrt{1 - \beta_\parallel^2}\,\gamma,$$

then it is straightforward to see that the solution of the equation is

$$\xi = \frac{\xi_0 + 1 + (\xi_0 - 1)e^{-t/\tau}}{\xi_0 + 1 - (\xi_0 - 1)e^{-t/\tau}},$$

where ξ_0 is the initial value of ξ. This solution shows that ξ decays to a value of 1, which implies that γ approaches $1/\sqrt{1 - \beta_\parallel^2}$, i.e., all the transverse energy radiates away leaving only the longitudinal energy. Of course, before all of the transverse energy is radiated the positron moves into the loss cone and exits the trap.

We can also estimate the confinement time of a particle in the trap. Using the exact solution to the energy equation leads to a complicated solution, so we will approximate the time dependence of ξ by

$$\xi \simeq 1 + (\xi_0 - 1)e^{-t/\tau},$$

in order to get a fairly simple answer. This function approximates the exact solution fairly well for all times. With this time dependence, it is straightforward to find that a particle will stay trapped for a time

$$t_{\text{trapped}} \simeq \frac{\tau}{\sqrt{1-\beta_{\parallel}^2}}\left(\ln\left(\sqrt{\frac{1-\beta_{\parallel}^2}{1-\beta_{\parallel}^2-\beta_{\perp}^2}} - 1\right)\right.$$
$$\left. - \ln\left(\sqrt{\frac{(R-1)(1-\beta_{\parallel}^2)}{1-\beta_{\parallel}^2 R}} - 1\right)\right).$$

Using a mirror ratio of 4 and a maximum positron energy of 1.74 MeV, we have numerically averaged the coefficient of τ over the trapped part of the Fermi distribution and found that the average confinement time is

$$< t_{\text{trapped}} > \simeq 3\tau.$$

Setting this time equal to the isotope decay time allows us to determine the magnetic field. For example, for a 124-sec decay time we find that the magnetic field required is about 3.5 kG. A 1.74-MeV positron has a gyroradius of 2 cm in such a field.

3.3 Output Distribution and Emittance

If we ignore collisions and the particles that are born in the loss cone, the output distribution of the trap can be computed from the initial distribution of positrons and the assumption that the positrons radiate their transverse energy until they hit the loss cone. The output distribution is concentrated along the loss cone in velocity space with a distribution in longitudinal velocity given by the initial longitudinal velocity distribution of trapped positrons.

The full initial distribution of positrons is just the Fermi distribution [4], i.e., the distribution function, $f(\vec{p})$, in momentum is

$$f(\vec{p}) \propto (\chi - \gamma)^2,$$

where χ is the isotope-dependent maximum positron γ and where we have ignored the normalization constant. Converting from momenta, $\vec{p}$, to dimensionless velocities, $\vec{\beta}$, we find that the distribution function in velocity is

$$g(\beta_{\|},\beta_{\perp}) \propto \beta_{\perp}\gamma^5(\chi-\gamma)^2,$$

where

$$\gamma = \frac{1}{\sqrt{1-\beta_{\|}^2-\beta_{\perp}^2}}.$$

The distribution function for trapped particles is zero outside the region

$$\frac{|\beta_{\|}|}{\sqrt{R-1}} \leq \beta_{\perp} \leq \sqrt{1-\frac{1}{\chi^2}-\beta_{\|}^2},$$

$$\beta_{\|}^2 \leq (\frac{R-1}{R})(1-\frac{1}{\chi^2}).$$

Integrating in $\beta_{\perp}$ over the allowed region for trapped particles we find that the distribution function in $\beta_{\|}$, $h(\beta_{\|})$, is

$$h(\beta_{\|}) \propto \frac{\chi^5}{15} - \frac{2\chi^2}{3D^{3/2}} + \frac{\chi}{D^2} - \frac{2}{5D^{5/2}},$$

where D is defined as

$$D \equiv 1 - \frac{\beta_{\|}^2 R}{R-1}.$$

For a particle exiting with a given value of $\beta_{\|}$ the transverse velocity is

$$\beta_{\perp} = \frac{|\beta_{\|}|}{\sqrt{R-1}},$$

and the total γ is

$$\gamma = (1 - \frac{\beta_{\|}^2 R}{R-1})^{-1/2}.$$

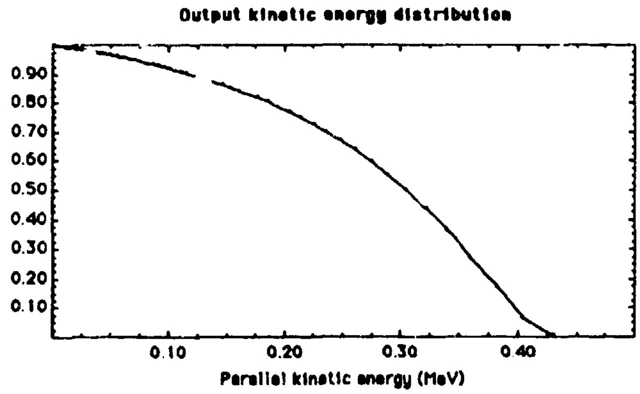

Figure 2

The distribution function in parallel kinetic energy is shown in Figure 2. For an initial maximum positron energy of 1.74 MeV, the energy of the positron from oxygen decay, the average output energy of the trapped particles is 124 keV. We can estimate the output emittance by noting that all the exiting particles have the same angle, $\theta = \tan^{-1}(1/\sqrt{2(R-1)})$, so that multiplying by the trap radius, r_t, the normalized emittance, ϵ_n, is approximately

$$\epsilon_n \sim \frac{\pi\gamma\beta r_t}{\sqrt{2(R-1)}}.$$

As an example, taking the γ and β appropriate for 124 keV particles and using $R = 4$ and $r_t = 2 - 4$ cm, we find $\epsilon_n \sim 6000 - 12000\pi$ mm mrad, i.e., about a factor of $4 - 8$ less than the emittance produced without a trap.

3.4 Trap Limits

Though the trap parameters seem to be fairly practical, an average field of $\sim$ 3.5 kG, a peak field of $\sim$ 14.0 kG, a radius of a few centimeters, we still have not specified the trap length. Knowing the number of positrons produced in an isotope decay time we know how many positrons have to be contained in the trap. Because we should minimize the trap radius in order to minimize the output emittance, the radius is set by the gyroradius. Thus the trap length is determined when we determine the maximum density that the trap will hold. The maximum trap density allowed is set by two limits, (1) the annihilation rate of the positrons in the trap, and (2) plasma effects.

3.4.1 Annihilation

Because the positron annihilation rate depends on plasma density, the requirement that the probability of annihilation be considerably less than one sets a limit on the plasma density in the trap. The annihilation rate, A, is given in Heitler [5] and, assuming a uniform background of electrons, is given by

$$A = \pi n_e r_0^2 c,$$

where r_0 is the classical radius of the electron, n_e is the electron density, c is the speed of light, and the rate is in $\sec^{-1}$. Assuming a decay time of 124 sec and requiring that the probability of decay be less than one, we find that the electron density must satisfy $n_e < 10^{12}$ cm^{-3}. As an example, let us assume a positron production rate of 10^{14} per second and a trap radius of 4 cm, and measure plasma densities in units of 10^{12} cm^{-3}, i.e., $n_e \equiv \bar{n}_e * 10^{12}$ cm^{-3}; we then find that the plasma length, L, is constrained by

$$L > \frac{2.5}{\bar{n}_e} \text{ meters.}$$

Taking $\bar{n}_e \sim 0.1$ in order to have a 90% probability of survival gives a trap length of ~ 25 meters for this particular isotope in this example. The isotopes with shorter decay times will require, of course, smaller traps.

3.4.2 Plasma Effects

In the trap, it is necessary that the magnetic field pressure be large enough to confine the plasma; this capability is measured by the plasma beta, defined as

$$\beta_p \equiv \frac{8\pi n T}{B^2},$$

where n is the plasma density, T is the plasma temperature, and B is the magnetic field. In principle, there should be a sum over particle species in this definition, but in our case the high temperature of the positrons will cause their pressure to dominate all other species. The condition for equilibrium to be possible is that $\beta_p < 1$. Using the parameters of the example, $B = 3.5$kG $n_p = 10^{11}\text{cm}^{-3}$, and taking the temperature, $T = 0.87$ MeV, i.e., about half the maximum positron energy, we find $\beta_p \approx 0.29$. Thus, equilibrium should be possible, but care must be taken to insure MHD stability for this high a value of beta. To decrease beta, we can either consider designs with greater lengths, or consider other targets with shorter decay times.

There is, of course, the possibility of many instabilities in such a plasma. The most obvious mirror mode, the loss cone instability [6], is actually helpful in this case because it tends to eject the background ions, along with a roughly equal number of free electrons, and thus reduce the annihilation rate; the low frequency of this mode means that it is unlikely to have much effect on the positrons. There may, of course, be instabilities that affect the positrons directly, but these are likely to be considerably ameliorated by large Larmor radius and wall effects. It thus seems that plasma effects in the trap may be manageable, especially if longer traps are considered, but clearly this must be verified by carrying out experiments.

3.5 Extraction

Controlled extraction of the positrons from the trap is relatively straightforward. By applying longitudinal RF accelerating fields at one end of the trap, we can easily extract the positrons preferentially from that end of the trap in phase with the RF. (To further favor extraction from one end, we could easily weaken the magnetic field on that end.) The depth of the RF bucket required, about 250 keV for the oxygen decay, is not excessive. By continuing the magnetic field in a solenoid configuration one could provide focusing for several cells of RF acceleration until the energy is high enough to allow AG focusing cells to be used.

4 Further Cooling

In the previous sections we have seen that it seems possible to construct a relatively straightforward trap that can put out high currents of positrons with normalized emittances in the range $\epsilon_n \sim 6000 - 12000\pi$ mm mrad. Though these emittances might be small enough to accept into a linac and subsequently a damping ring, it would be easier to design the damping rings if the emittances were lower by another factor of 5 or so. Of course, we can use electrostatic fields to further confine the positrons in the trap and obtain even lower emittances, but it is useful to examine other alternatives as well. In this section we will show that foil cooling should work and that the required foils and their heat loads are reasonable. It is then possible to consider a scheme in which the positron cooling is done in three stages: first, radiative cooling in the magnetic trap decreases the emittances by a factor of 4 or so; then foil cooling in a linac decreases the emittances by another factor of about 5; finally, damping rings reduce the emittances to the required final values.

Foil cooling for applications in accelerators has been proposed and studied before [7], [8], [9], [10]. Briefly, the idea is that a plane foil in the beam line causes the particles to lose energy parallel to their trajectories; if the foil is followed immediately by an accelerating gap the particles regain the lost energy, but only in the direction parallel to the beam axis; thus they are cooled transversely. If the kinetic energy of the particles is greater than about three times their rest mass, the particles are cooled longitudinally as well, though for nuclear and particle- physics applications we do not require longitudinal cooling. Of course, the positrons are also heated by the scattering because the RMS scattering angle with respect to the direction of propagation is not zero. The minimum emittance that can be achieved is thus determined by the balance between these two processes. The performance of the foil cooling system can also be limited by positron annihilation and by the heat load on the foils. For the examples we consider, we will see that annihilation and foil heat load are not major concerns.

For the transverse degrees of freedom the equation for either emittance, ϵ, assuming thin foils is [9]

$$\frac{d\epsilon}{dn} = -\frac{dE}{dz}\frac{\delta}{E}\epsilon + \frac{\beta_f \theta^2_{\rm rms}}{2},$$

where δ is the foil thickness, n is the variable numbering the cooling cell, β_f is the beta function at the foil location, and $\theta_{\rm rms}$ is the RMS scattering angle in the foil.

For positrons with energies between 5 and 10 MeV the predominant energy loss mechanism is multiple scattering and the energy loss formula is approximately [1]

$$\frac{dE}{dz} = \frac{4\pi n Z e^4}{mv^2}\left(\ln((\gamma-1)\sqrt{\frac{\gamma+1}{2}}\frac{mc^2}{\hbar<\omega>}) - \frac{v^2}{c^2}\right),$$

where E is the energy of the proton, n is the density of the target, Z is the number of protons in the target nucleus, m is the electron mass, M is the proton mass, and $\hbar<\omega>$ is an average excitation energy of the target atoms. As an example, if we assume the positrons are at 5 MeV and the foils are carbon, then the parameters are

$$n \simeq 10^{23}\ \mathrm{cm}^{-3},$$

$$Z = 6,$$

$$\hbar<\omega> \simeq 60\ \mathrm{eV},$$

and

$$\gamma \simeq 10.8.$$

We then find that

$$\frac{1}{E}\frac{dE}{dz} \simeq 0.69 \text{ cm}^{-1}.$$

A foil of this type that would induce a 5% energy loss would then be about 0.072 cm thick. If the positron current were 100 μA, the heat deposited would be about 25 W. Cooling such a heat load would be fairly easy. Ignoring for the moment the heating, it would take about 46 foils of this type followed by RF gaps imparting 250 kV each to to the positrons to reduce the emittances by a factor of 10.

The expression for θ_{rms} can also be obtained from reference [1] and is

$$\theta_{\text{rms}} \simeq \frac{16\pi n Z^2 e^4}{\gamma^2 m^2 v^4} \ln(\frac{210}{Z^{1/3}})\delta.$$

Balancing the damping against the heating we see that the minimum emittance that can be obtained is

$$\epsilon_{\min} \simeq \frac{2\beta_f Z(\gamma-1)c^2}{\gamma^2 v^2} \frac{\ln(\frac{210}{Z^{1/3}})}{\left(\ln((\gamma-1)\sqrt{\frac{\gamma+1}{2}}\frac{mc^2}{\hbar<\omega>}) - \frac{v^2}{c^2}\right)}.$$

As an example, if we use the previous parameters, we find

$$\epsilon_{\min} \simeq 0.43\beta.$$

At 5 MeV it should be possible design a transport channel with $\beta_f \simeq 0.5$ cm at the foil positions so that the minimum emittance achievable would be about 2150 π mm mrad, a value small enough to easily accept in a damping ring. Note that the β_f function need not be this small in the initial foils; when the beam is hot the beta function can be larger because heating is less important. The beta function at the foils can be reduced as the beam cools in order to continue the cooling process.

If we double the energy of the positrons to 10 MeV, assuming the same β_f function, then we could obtain emittances about a factor of two lower. However, the heat load on the foils would double, the RF gaps would have to supply twice the acceleration, and it would be harder to achieve this low a β_f function. At even higher energies, bremsstrahlung would begin to compete with scattering as the dominant energy loss mechanism and introduce different scalings. It thus seems that cooling should take place at about 5 to 10 MeV.

To estimate the annihilation probability, P, we use the cross section from reference [5] to find that

$$P \simeq LnZ\pi r_0^2 \frac{1}{\gamma + 1}\left(\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln(\gamma+\sqrt{\gamma^2-1}) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right),$$

where L is the total thickness of foils traversed and r_0 is the classical electron radius. Assuming 5-MeV positrons traverse 50 foils each 0.072 cm thick, we find that the total annihilation probability is about 0.14; thus annihilation is not a problem in this example.

5 Summary

We have presented a scheme that would allow the use of low energy proton beams to produce substantial fluxes of useful positrons for nuclear and high-energy physics applications. With a better understanding of the magnetic traps required it might also be possible to produce very high fluxes of cold positrons for solid-state diagnostics applications.

The principal uncertainty in our scheme is estimeated to be the trap physics, which primarily affects the overall trap size. However, even with fairly conservative assumptions on trap performance, it seems that it should be possible to design traps with reasonable overall sizes.

6 Acknowledgments

At various stages of this work we have benefited from conversations with Rich Sheffield, Dave Neuffer, and Fred Wysocki. In particular, Dave Neuffer urged that we consider the foil cooling scheme and Rich Sheffield suggested that we consider RF extraction from the trap.

References

[1] J. D. Jackson, *Classical Electrodynamics, 1st ed.* (John Wiley and Sons, Inc., New York, 1962).

[2] D. B. Cline, Proceedings of the Workshop on Advanced Accelerator Concepts, Madison, 1986, AIP Conference Proceedings #156, 435 (1987).

[3] G. Gibson, W. C. Jordan, E. J. Lauer, Phys. Rev. Lett. 141, **5**, Aug. 15, 1960.

[4] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics.* (Springer-Verlag, Berlin, 1979).

[5] W. Heitler, *The Quantum Theory of Radiation, 3d ed.* (Oxford University Press, 1954).

[6] N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics.* (McGraw-Hill Book Company, New York, 1973).

[7] E. A. Perevedentsev and A. N. Skrinsky, Proc. Second ICFA Workshop on Accelerators and Detectors, 61 (1979).

[8] V. V. Parkhomchuk and A. N. Skrinsky, Proc. 12th Int. Conf. on High-Energy Accelerators, 485 (1983).

[9] D. Neuffer, Particle Accelerators **14**, 75 (1983).

[10] D. Neuffer, Proc. 12th Int. Conf. on High-Energy Accelerators, 481 (1983).